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Attaining a good primal solution to the uncapacitated transportation problem

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Abstract: Transportation of products from sources to destinations with minimal total cost plays an important role in logistics and supply chain management. The Uncapacitated Transportation Problem (UTP) is a special case of network flow optimization problem. The prime objective of this UTP is to minimize the total cost of transporting products from origins to destinations subject to the respective supply and demand requirements. The UTP consists of special network structure. Due to the special structure of this problem, the transportation algorithm is preferred to solve it. The transportation algorithm consists of two major steps: 1) Finding an Initial Feasible Solution (IFS) to TP and 2) Examining the optimality of this IFS. A better IFS generates a lesser number of iterations to obtain a Minimal Total Cost Solution (MTCS). Recently, Juman and Nawarathne (2019)'s Method was introduced to find an IFS to UTP. In this paper, the Juman and Nawarathne (2019)'s Method is improved to get a better IFS to a UTP. A comparative study on a set of benchmark instances illustrates that the new improved method provides better primal solutions compared to the Juman and Nawarathne (2019)'s Method. The proposed method is found to yield the minimal total cost solutions to all the benchmark instances.

Keywords: logistics and supply chain management, uncapacitated transportation problem, supply and demand requirements, primal solution, minimal total cost solution.

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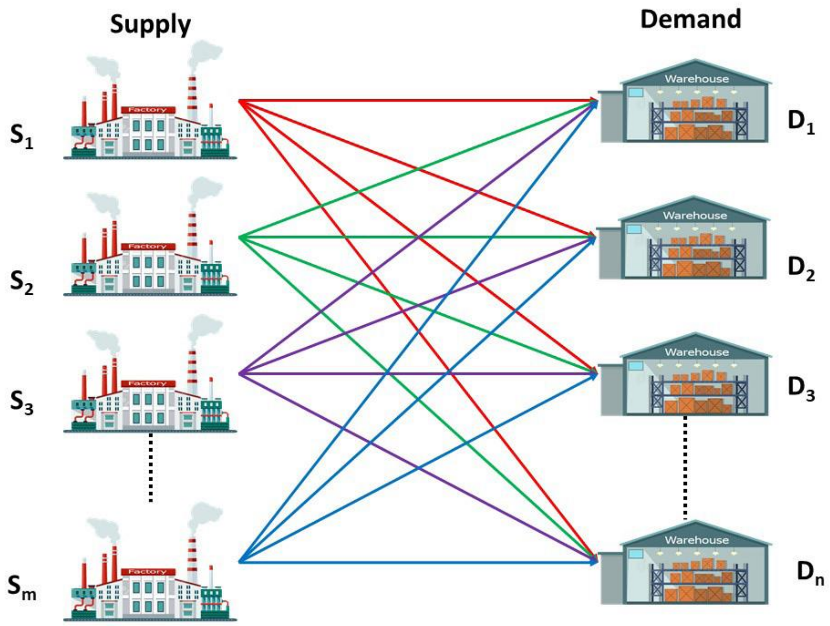
1. Introduction

The transportation problem is a special type of network flow problem where the objective is to minimize the cost of distributing a product from a number of sources or origins to a number of destinations.The general transportation problem deals with a set of supply nodes $(S_1, S_2, \dots, S_m)$ and set of demand nodes $(D_1, D_2, \dots, D_n)$. The data consists of availability of each supply node $(s_1, s_2, \dots, s_m)$, the requirement at each demand node $(d_1, d_2, \dots, d_n)$, and the cost of shipping a unit from each supply node to each demand node (c_{ij}). The quantity to be transported from each supply node to each demand node (x_{ij}) is determined in this problem. Network representation of this problem (taken from Juman & Nawarathne, 2019) is provided in Fig.1.

1.1. Structure of the transportation problem

Figure 1: Network representation of classical transportation problem



Ample literature exists on different variants of the transportation model and their initial solution methods with fixed demands and supplies (ex. Deshmukh, 2012; Sharma & Sharma, 2000; Juman & Hoque, 2015; Sharma & Prasad, 2003; Juman et al. 2013a; Juman & Perera, 2015; Juman et al. 2013b; Ekanayake et al., 2020; Ekanayake et al., 2021). Recently, Juman and Nawarathne (2019) proposed an efficient alternative method for solving transportation problems. This paper improves this existing method to attain a better efficient solution to the problem.

1.2. Mathematical formulation of a general transportation problem

Notations

m: Total number of supply nodes (suppliers)

n : Total number of demand nodes (buyers)

x_{ij}:Number of units transported from i^{th}supply node to j^{th}demand node

c_{ij}:Unit transportation cost from i^{th}supply node to j^{th} demand node

s_i: Supply quantity (in units) for i^{th}supply node

d_j: Demand quantity (in units) for j^{th} demand node

Where $i = 1,2,3, \dots, m$ and $j = 1,2,3, \dots, n$

Mathematical Model

$$Min \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij}$$

Subject to

$$\sum_{j=1}^{n} x_{ij} \le s_i \quad , \qquad i = 1,2, \dots ,m$$

$$\sum_{i=1}^{m} x_{ij} \ge d_j \quad , \qquad j = 1,2, \dots ,n$$

Where $x_{ij} \ge 0 \;\; \forall\, i,j$.

A necessary and sufficient condition for the existence of a feasible solution to the transportation problem is, the total supply quantity should be equal to total demand quantity.

$$\sum_{i=1}^{m} s_i = \sum_{j=1}^{n} d_j$$

If total supply quantity is equal to total demand quantity, then the transportation problem is called as a balanced transportation Problem. If not the problem is called as an unbalanced transportation problem.

1.3. Minimal total cost solution to the transportation problem

The transportation problem consists of a special structure. Due to the special structure of the problem, a specialized algorithm named Transportation Algorithm is used to obtain the minimal total cost solution to it. The Transportation algorithm consists with two steps.

Step 1: Determine an initial feasible solution (IFS) for the problem.

Step 2: Test for optimality of the IFS.

1.4. Methods of finding IFS to the transportation problem

There are various methods to find the IFS for a transportation problem. Few of the best available ones are provided here.

- North – West Corner Rule (2004)
- Minimum Cost Method (2004)
- Vogel's Approximation Method (1958)
- Juman and Hoque Method (2015)
- Juman and Nawarathne Method (2019)

After finding the Initial Feasible Solution (IFS) for a transportation problem, Minimum Total Cost Solution (MTCS) can be obtained by using Stepping Stone Method (SSM) or Modified Distribution Method (MODI).

Reminder of this paper is as follows: Section 2 presents the existing approach and its improvement. Results and discussions are performed in section 3. Finally, concluding remarks along with future extensions are drawn in section 4.

2. Methodology

2.1. Juman and Nawarathne (2019)'s Method

An existing method: *Juman & Nawarathne (2019)'s method*

Step 1: Check whether the problem is balanced or not. If it is unbalanced, then add dummy demander or dummy supplier to make the problem balanced with zero transportation cost.

Step 2: For each row of the transportation matrix, identify the least cost cell. Assign the respective supply quantities there.

Step 3: For assigned allocation in each of columns (without taking into account any crossed column, if exist) check whether the column sum is less than or equal to the respective demand quantity. If so, go to step 10.

Step 4: For each of the allocations in an unmet column considering the row containing that allocation determine the difference between the second least and the least unit cost, and identify the smallest of them (in case of tie, identify the smallest with the largest unit cost) Else identify the smallest difference for each of them separately and go to step 5.

Step 5: Check whether there exist a cell (or cells) in an unmet column not containing the second least unit cost corresponding to the smallest of the difference between the second least and the least unit cost in a row for each of the allocation in an another unmet column. If such a column exists, identify the former unmet column go to step 8

Step 6: Pick up any two unmet columns. For each of them, find differences between the 2nd least and the least unit cost of that unmet column.

Let the smallest of the differences for an unmet column corresponds to the least unit cost c_1, and the smallest of the differences for the other unmet column corresponds to the smallest unit cost e_1. Let c_1, $c_2$andc_3 be the 1st, 2ndand 3rd least unit costs in a row ande_1, $e_2$and $e_3$be the 1st, 2ndand 3rdleast unit cost in another row.

If $(c_3 - c_1) > (e_3 - e_2)$, then identify the unmet column containing the least unit cost c_1. Else identify the other unmet column (containing the least unit cost e_1)

Step 7: Pick up any two unmet column for each of them. If there is no such connection between the two columns, then select the column with minimum difference. And go to step 8

Step 8: Considering the identified unmet column in step 4, 5, 6 or 7 and corresponding to the smallest of the differences between the second least and the least unit cost in a row for each of the allocations in this column, transfer the maximum possible amount of excess demand quantity which can make next column met, from the least unit cost cell to the next least unit cost cell in a row. (If the demand value is smaller than the value of the allocation in the unmet column, transfer the full amount of the allocation)

If there remains more excess demand in the selected column, do the same for the next smallest difference of the second least and least unit costs and continue the transferring process until the selected unmet column become met.

Step 9: Cross of the column that has completely been satisfied by removal of excess demand quantity just made, and go to step 3

Step 10: Stop, and take the current solution as the initial feasible basic solution.

2.2. A modified method to find an IFS for a TP

In this section, a modification is introduced to the existing *Juman and Nawarathne (2019)'sMethod* (JNM) to obtain an improved IFS to a balanced transportation problem. The steps of Modified Method (MM) are described below:

Algorithm for modified method

An improved efficient alternative method: Heuristic for obtaining an efficient IFS to the transportation problem

Step 1: Apply the steps 1-4 of the *Juman and Nawarathne (2019)'s Method*.

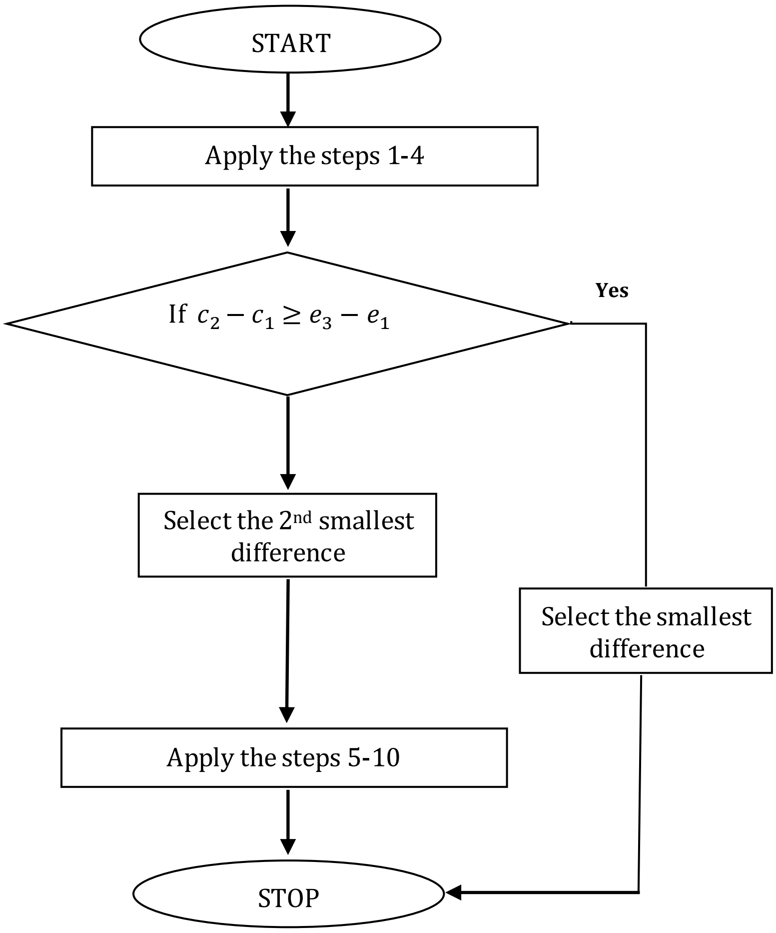
Step 2: Let the smallest of the differences corresponds to the least unit cost $c_1$and the second smallest of the differences corresponds to the smallest unit cost e_1.Let c_1, c_2 and c_3 be the 1st, 2nd and 3rd least unit costs in a row and e_1, e_2 and e_3 be the 1st, 2nd and 3rd least unit cost in another row.

If $c_2 - c_1 \geq e_3 - e_1$, then select the smallest difference. Else select the 2nd smallest difference and go to Step 5.

Step 3: Apply the steps 5-10 of the *Juman and Nawarathne (2019)'s Method*.

The flow chart of the above new modified method is also provided below.

Figure 2: Flow chart representation of the above new modified method



3. Result and discussion

Modified Method of this paper is compared against the existing methods, Vogel's Approximation Method (VAM) (1958), Juman and Hoque (2015)'s Method (JHM), Juman and Nawarathne (2019)'s Method(JNM). Comparison among these four methods is performed using the solutions of 16 benchmark instances. The Stepping Stone Method (SSM) is used to obtain Minimal Total Cost Solution (MTCS) starting from the initial feasible solution (IFS) which is obtaining from VAM, JHM, JNM or Modified Method (MM).

The performance measure and the comparison on the iteration numbers of these four methods are provided in Table 1-4 and Figures 3-8.

3.1. Performance measure of modified method (MM) over VAM, JHM and JNM

A performance measure of our modified method of this paper is shown in the following Tables 1 and 2 along with Figures 3 and 4.

Table 1: Performance measure of MM over VAM, JHM and JNM for 9 benchmark instances

Problem chosen from	Initial cost with an IFS by				% decrease in TCIFS by MM over VAM, JHM and NM			Minimal Total Cost Solution (by Lingo)	% increase from the minimal total cost by			
	VAM	JHM	JNM	MM	VAM	JHM	JNM		VAM	JHM	JNM	MM
Srinivasan and Thompson	955	880	880	880	7.85	0.00	0.00	880	8.52	0.00	0.00	0.00
Sen*et al.*	2,164,000	2,146,750	2,146,750	2,146,750	0.80	0.00	0.00	2,416,750	0.80	0.00	0.00	0.00
Deshmukh	779	743	743	743	4.62	0.00	0.00	743	4.85	0.00	0.00	0.00
Ramadan and Ramadan	5,600	5,600	5,600	5,600	0.00	0.00	0.00	5,600	0.00	0.00	0.00	0.00
Kulkarni and Datar	880	840	840	840	4.55	0.00	0.00	840	4.76	0.00	0.00	0.00
Schrenk*et al.*	59	59	59	59	0.00	0.00	0.00	59	0.00	0.00	0.00	0.00
Samuel	28	28	28	28	0.00	0.00	0.00	28	0.00	0.00	0.00	0.00
Imam *et al.*	475	460	435	435	8.42	5.43	0.00	435	9.20	5.75	0.00	0.00
Adlakha and Kowalski	390	390	390	390	0.00	0.00	0.00	390	0.00	0.00	0.00	0.00

Table 2: Performance measure of MM over VAM, JHM and JNM for 7 benchmark instances from Juman and Hoque (2015)

Problem No.	Initial cost with an IFS by				% decrease in TCIFS by MM over VAM, JHM and NM			Minimal Total Cost Solution (by Lingo)	% increase from the minimal total cost by			
	VAM	JHM	JNM	MM	VAM	JHM	JNM		VAM	JHM	JNM	MM
1	5,125	4,525	4,525	4,525	11.71	0.00	0.00	4,525	13.26	0.00	0.00	0.00
2	3,520	3,460	3,460	3,460	1.70	0.00	0.00	3,460	1.73	0.00	0.00	0.00
3	960	920	920	920	4.17	0.00	0.00	920	4.35	0.00	0.00	0.00
4	849	809	809	809	4.71	0.00	0.00	809	4.94	0.00	0.00	0.00
5	465	417	465	**417**	10.32	0.00	10.32	417	11.51	0.00	11.51	0.00
6	3,663	3,458	3,458	3,458	6.00	0.00	0.00	3,458	5.93	0.00	0.00	0.00
7	109	109	109	109	0.00	0.00	0.00	109	0.00	0.00	0.00	0.00

Figure 3: Bar chart of initial cost vs. benchmark instances in case of solving by VAM, JHM, JNM and MM

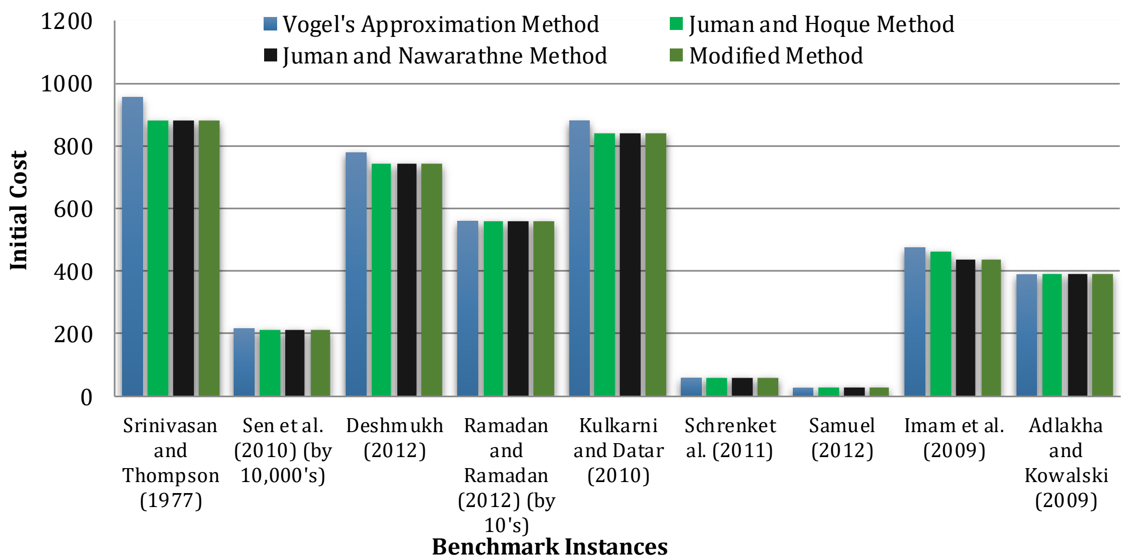
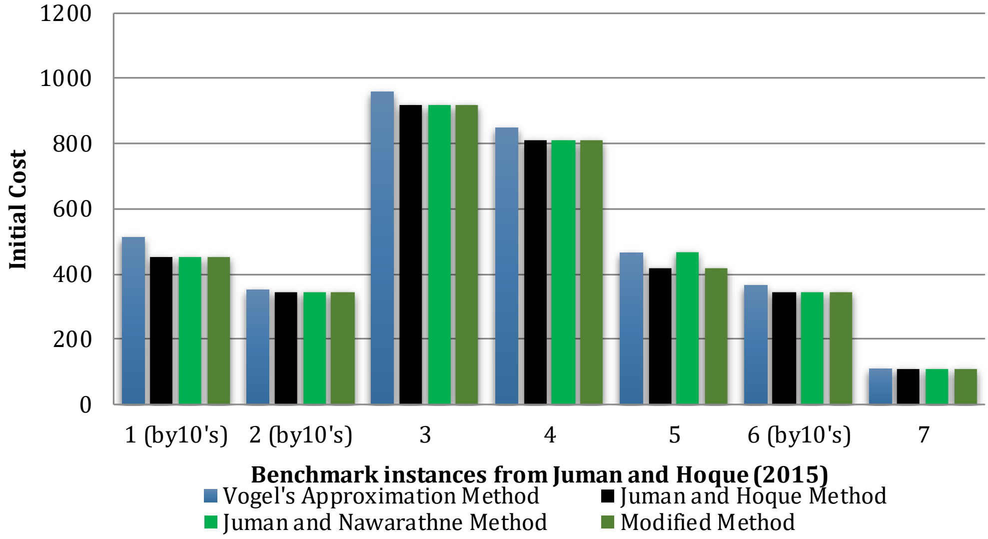


Figure 4: Bar chart of initial cost vs. 7 benchmark instances from Juman and Hoque (2015) in case of solving by VAM, JHM, JNM and MM.



3.2. Comparison on the iteration numbers

Comparative assessment on the iteration numbers is also shown in Tables 3-4 and Figs. 7-8 below. Besides the percentage improvements to the new modified method are shown as bar chart plots in Figs. 5 and 6.

Table 3: Comparative study on the iteration number taken by SSM coupled with MM, VAM, JHM and JNM for 9 benchmark instances

Problem Chosen from	Iterations taken by SSM to obtain MTCS by starting with an IFS obtained by			
	VAM	JHM	JNM	MM
Srinivasan and Thompson (1977)	1	0	0	0
Sen*et al.* (2010)	2	0	0	0
Deshmukh (2012)	1	0	0	0
Ramadan and Ramadan (2012)	0	0	0	0
Kulkarni and Datar (2010)	2	0	0	0
Schrenk*et al.* (2011)	0	0	0	0
Samuel (2012)	0	0	0	0
Imam *et al.* (2009)	1	1	0	0
Adlakha and Kowalski (2009)	0	0	0	0

Table 4: Comparative study on the iteration number taken by SSM coupled with MM, JHM and JNM for 7 benchmark instances from Juman and Hoque (2015)

Problem No.	Iterations taken by SSM to obtain MTCS by starting with an IFS obtained by			
	VAM	JHM	JNM	MM
1	1	0	0	0
2	2	0	0	0
3	1	0	0	0
4	1	0	0	0
5	1	0	1	0
6	2	0	0	0
7	0	0	0	0

Figure 5: Percentage decreases in total cost for IFS by MM over VAM, JHM and JNM for 9 benchmark instances

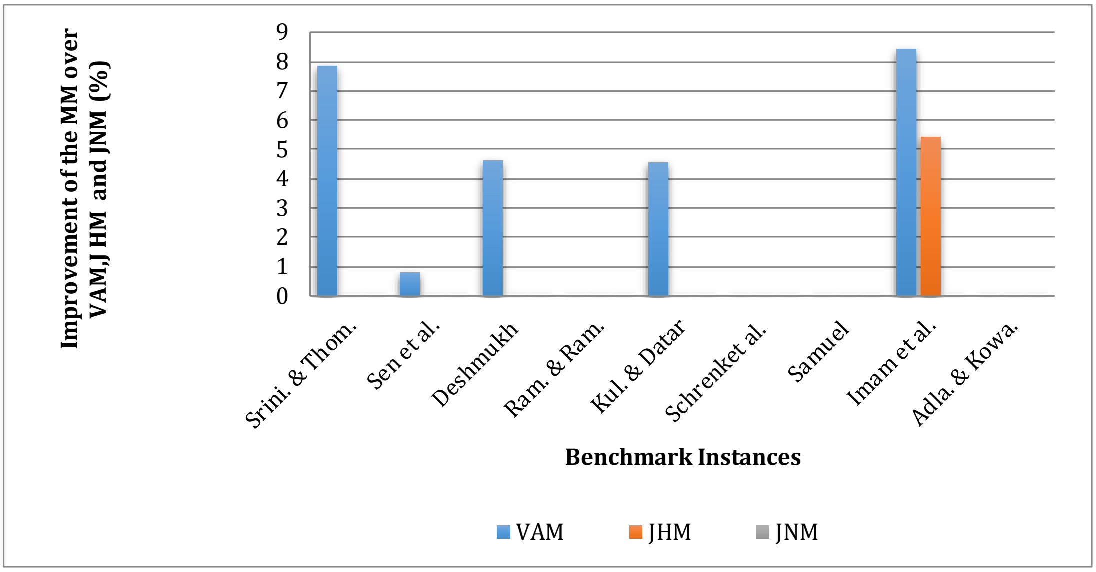


Figure 6: Percentage decreases in total cost for IFS by MM over VAM, JHM and JNM for 7 benchmark instances from Juman and Hoque (2015)

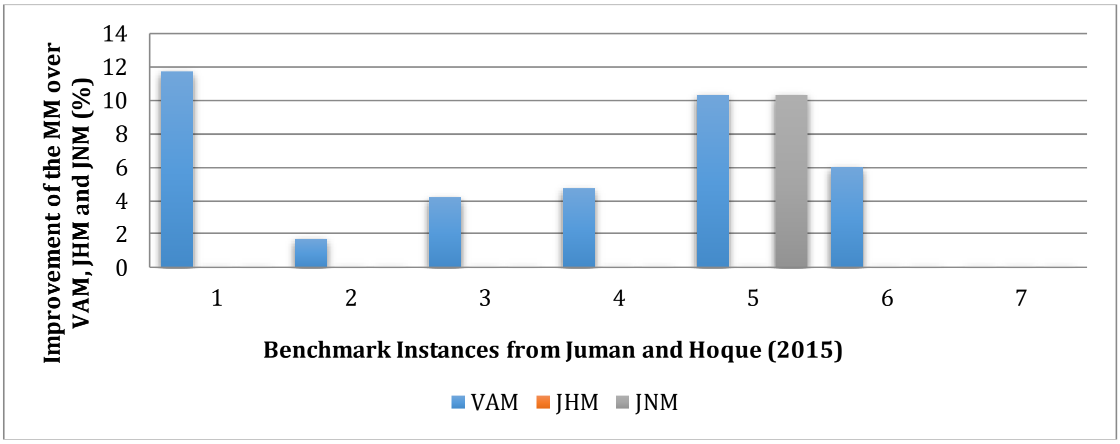


Figure 7: Number of Iterations taken by SSM to obtain MTCS by starting with an IFS obtained by VAM, JHM, JNM and MM for 9 benchmark instances

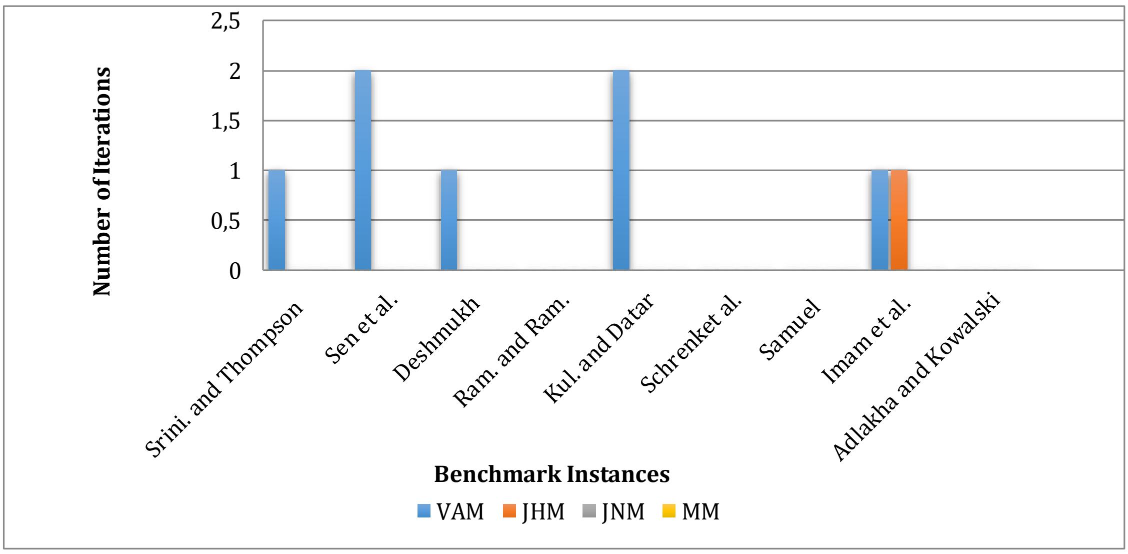
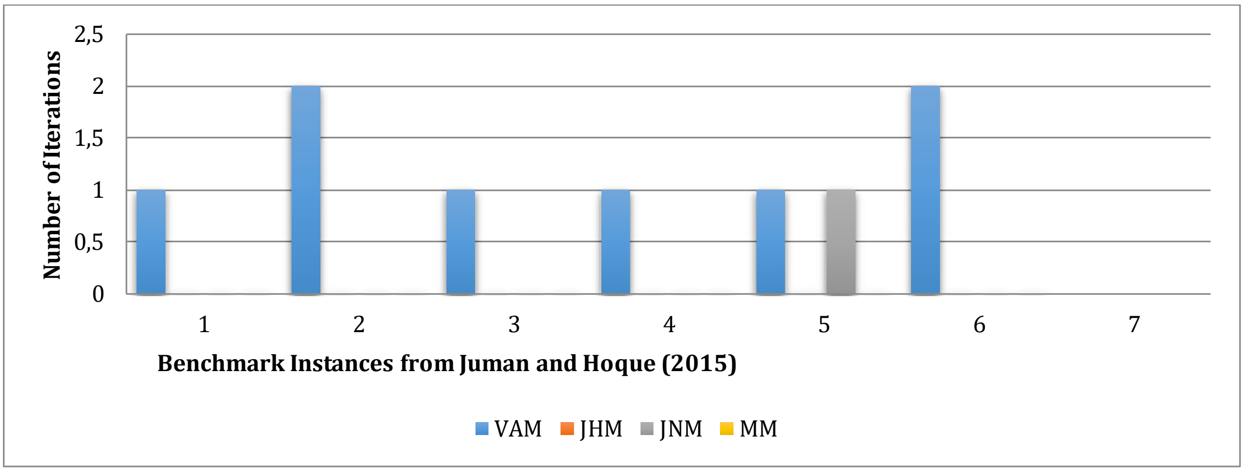


Figure 8: Number of Iterations taken by SSM to obtain MTCS by starting with an IFS obtained by VAM, JHM, JNM and MM for 7 benchmark instances from Juman and Hoque (2015)



It can be easily noticed from the Tables 1-4 and Figures 3-8 that the modified method produces the same or better IFS to all considered benchmark instances compared to all the existing methods. Consequently, the modified method can be considered as an alternative method to obtain the IFS to a transportation problem.

4. Conclusion and future scope

To obtain a minimal total cost solution to a transportation problem, finding an initial feasible solution is the prime requirement. According to the initial feasible solution of the problem, the number of iterations to get the minimal total cost solution can be changed. If an initial feasible solution becomes closer to the minimum total cost, the minimal total cost solution can be obtained with lesser number of iterations. There are several methods to find an initial feasible solution to a transportation problem. Among them, Vogel's Approximation Method (1958), Juman and Hoque (2015)'s Methodand Juman and Nawarathne (2019)'s Method can be considered as the state of the art initial solution providers available in the literature. In this work, a new modification is introduced to attain an efficient initial feasible solution to the transportation problems. A comparative study illustrates that the Vogel's Approximation Method provides minimal total cost solution as the initial feasible solution only to 5 out of 16 benchmark instances. Furthermore, Juman and Hoque (2015)'s Method and Juman and Nawarathne (2019)'s Method give the minimal total cost solution as the initial feasible solution to 15 out of 16 benchmark instances. However, for all the 16 benchmark instances including that remaining problem the modified method provides same or better near-optimal cost solution compared with the existing ones. Thus, our new modified method outperform the existing Vogel's Approximation Method (1958), Juman and Hoque (2015)'s Method, Juman and Nawarathne (2019)'s Method.

Future research might be carried out to develop this modified method for the case of large number of origins and destinations. Throughout this research supply and demand quantities are all fixed. But in real world supply and demand quantities may vary. This research considers linear transportation problems. But the transportation problem can become nonlinear also. In those cases the current method cannot be applied. Thus the new modified method can be extended to include those variations. We intend to devote ourselves in this direction of future research.

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Conflicts of interest/Competing interests

The authors declare that they have no conflict of interest.

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